इंटरनेट मानक

## Disclosure to Promote the Right To Information

Whereas the Parliament of India has set out to provide a practical regime of right to information for citizens to secure access to information under the control of public authorities, in order to promote transparency and accountability in the working of every public authority, and whereas the attached publication of the Bureau of Indian Standards is of particular interest to the public, particularly disadvantaged communities and those engaged in the pursuit of education and knowledge, the attached public safety standard is made available to promote the timely dissemination of this information in an accurate manner to the public.

"जानने का अधिकार, जीने का अधिकार"
Mazdoor Kisan Shakti Sangathan
"The Right to Information, The Right to Live"

"पुराने को छोड नये के तरफ"
Jawaharlal Nehru
"Step Out From the Old to the New"

IS 7962 (1975): Methods of measurement for piezoelectric vibrations operating over the frequency range up to 30 MHz [LITD 5: Semiconductor and Other Electronic Components and Devices]

Act Number 22 of 2005
The Indian Parliament
RIGHT TO INFORMATION

"ज्ञान से एक नये भारत का निर्माण"
Satyanarayan Gangaram Pitroda
"Invent a New India Using Knowledge"

Made Available By
Public.Resource.Org

"ज्ञान एक ऐसा खजाना है जो कभी चुराया नहीं जा सकता है"
Bhartṛhari—Nītiśatakam
"Knowledge is such a treasure which cannot be stolen"

BLANK PAGE

BRUTUM FULMEN

IGNORANTIA JURIS NON EXCUSAT

PROTECTED BY COPYRIGHT

IS : 7962 - 1975
( Reaffirmed 2006 )

# *Indian Standard*

# METHODS OF MEASUREMENT FOR PIEZOELECTRIC VIBRATORS OPERATING OVER THE FREQUENCY RANGE UP TO 30 MHz

UDC 621·372·412 : 621·373·5 : 621·317·34

© *Copyright* 1976

INDIAN STANDARDS INSTITUTION
MANAK BHAVAN, 9 BAHADUR SHAH ZAFAR MARG
NEW DELHI 110002

Price Rs 10.00 Gr 6

*August* 1976

# *Indian Standard*

# METHODS OF MEASUREMENT FOR PIEZOELECTRIC VIBRATORS OPERATING OVER THE FREQUENCY RANGE UP TO 30 MHz

Piezoelectric Devices for Frequency Control and Selection Sectional Committee, ETDC 51


| | |
|---|---|
| *Chairman* | *Representing* |
| SHRI S. KUMAR | Bharat Electronics Ltd, Bangalore |
| *Members* | |
| SHRI S. P. BHIDE | Philips India Ltd, Bombay |
| SHRI A. DAS GUPTA (*Alternate*) | |
| DR V. N. BINDAL | National Physical Laboratory (CSIR), New Delhi |
| SHRI JOHN FRANCIS | Directorate General of Posts and Telegraphs, New Delhi |
| SHRI A. N. GARUD | Bhabha Atomic Research Centre, Bombay |
| DR J. D. JAIN | Central Electronics Engineering Research Institute (CSIR), Pilani |
| SHRI R. S. KALE | Directorate of Coordination (Police Wireless), Ministry of Home Affairs, New Delhi |
| SHRI M. S. SWAMINATHAN (*Alternate*) | |
| SHRI B. S. NARGAS | Wireless Planning & Coordination, Ministry of Communications |
| SHRI J. PATTABIRAMAN | Directorate General of Civil Aviation (Department of Civil Aviation) |
| SHRI B. R. CHATURVEDI (*Alternate*) | |
| RESEARCH ENGINEER | Directorate General of All India Radio, New Delhi |
| DR P. SAHA | Central Glass & Ceramic Research Institute (CSIR), Calcutta |
| SHRI ANNAMALAI (*Alternate*) | |
| SHRI M. SANKARALINGAM | Directorate General of Supplies and Disposals (Inspection Wing) |
| SHRI G. R. BHATIA (*Alternate*) | |
| SHRI N. S. SATHYANARAYANAN | Indian Telephone Industries Ltd, Bangalore |
| SHRI H. S. ANANTHANARAYANA RAO (*Alternate*) | |
| FT LT S. C. SHARMA | Ministry of Defence |
| SHRI K. PADMANABHAN (*Alternate*) | |
| SHRI N. SRINIVASAN, Director (Electronics) | Director General, ISI (*Ex-officio Member*) |

*Secretary*

SHRI P. K. BISWAS
Assistant Director (Electronics), ISI



© *Copyright* 1976

INDIAN STANDARDS INSTITUTION

**This publication is protected under the *Indian Copyright Act* (XIV of 1957) and reproduction in whole or in part by any means except with written permission of the publisher shall be deemed to be an infringement of copyright under the said Act.**

# *Indian Standard*

# METHODS OF MEASUREMENT FOR PIEZOELECTRIC VIBRATORS OPERATING OVER THE FREQUENCY RANGE UP TO 30 MHz

## 0. FOREWORD

**0.1** This Indian Standard was adopted by the Indian Standards Institution on 24 December 1975, after the draft finalized by the Piezoelectric Devices for Frequency Control and Selection Sectional Committee had been approved by the Electrotechnical Division Council.

**0.2** The object of this standard is to provide methods of measurement for piezoelectric vibrators operating over the frequency range up to 30 MHz.

**0.2.1** The transmission method described in this standard is suitable for frequencies up to 30 MHz for the commonly encountered ranges of the capacitance ratio $r$ and the figure of merit $M$, provided that errors due to instrumentation are taken into account. The equations presented in this standard have been formulated to correct these errors.

**0.3** In preparing this standard assistance has been derived from the IEC Pub 302-1969 'Standard definitions and methods of measurement for piezoelectric vibrators operating over the frequency range up to 30 MHz' issued by the International Electrotechnical Commission.

**0.4** In reporting the result of a test or analysis made in accordance with this standard, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS : 2-1960.*

## 1. SCOPE

**1.1** This standard describes the methods of measurement for piezoelectric vibrators operating over the frequency range up to 30 MHz.

## 2. TERMINOLOGY

**2.0** For the purpose of this standard, the definitions of terms given in IS : 1885 (Part V)-1965† and IS : 1885 (Part XXXIII)-1972‡ shall apply.

## 3. PIEZOELECTRIC VIBRATOR

**3.1 General** — A vibrator consists of an element usually in the form of a

*Rules for rounding off numerical values (*revised*).
†Electrotechnical vocabulary: Part V Quartz crystals.
‡Electrotechnical vocabulary: Part XXXIII Piezoelectric filters.

plate, bar or ring cut from a piezoelectric material and has electrodes attached to or supported near the element to excite one of its resonance frequencies.

**3.2 Equivalent Electric Circuit of a Piezoelectric Vibrator** — The electrical behaviour of a lightly damped mechanical vibrating system which is excited piezoelectrically through electrodes forming a two-terminal network can be represented in the vicinity of any mechanical resonance by an equivalent electric circuit (*see* Fig. 1) which consists of a capacitance $C_1$, inductance $L_1$ and resistance $R_1$ in series, shunted by the parallel capacitance $C_0$. The parameters are independent of frequency for isolated modes of motion.

NOTE — Generally, the mode in question is sufficiently isolated from other modes to permit this assumption. When this is not true, the equations and measuring methods outlined herein do not apply.

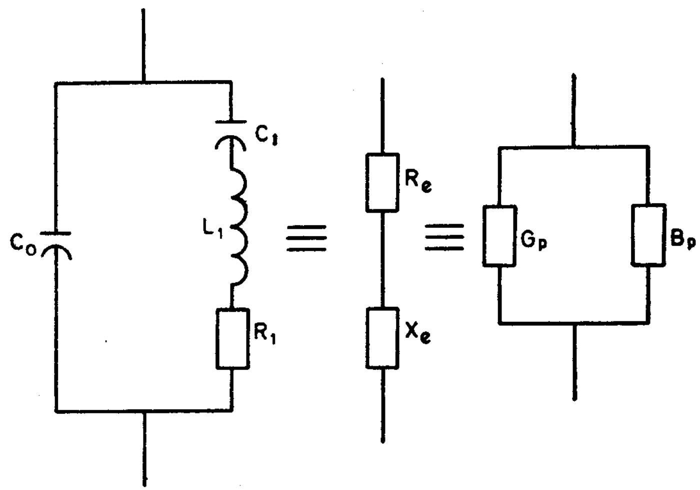



FIG. 1 EQUIVALENT ELECTRIC CIRCUIT OF A PIEZOELECTRIC VIBRATOR NEAR A RESONANCE

**3.3 Parameters of Piezoelectric Vibrators**

**3.3.1** The fundamental parameters $C_1$, $L_1$, $R_1$ and $C_0$ define the equivalent electric circuit shown in Fig. 1 and all other parameters may be derived from them. At a given frequency the parameters of the equivalent electric circuit generally approach constant values as the amplitude of vibration approaches zero. The amplitude which can be tolerated before the parameters are appreciably affected varies widely among vibrators of

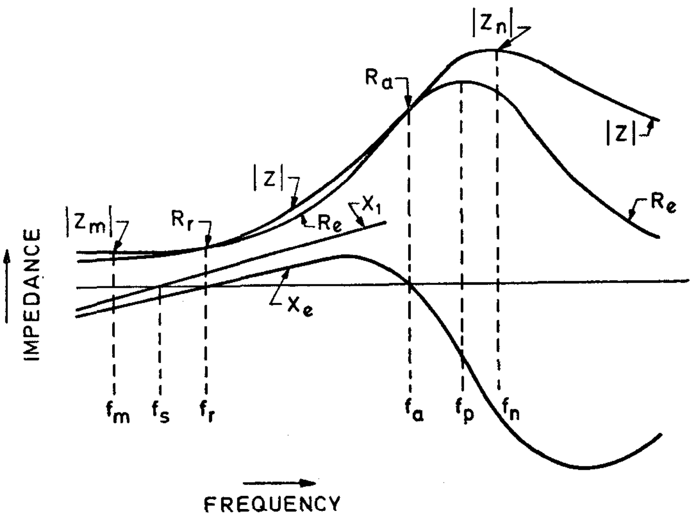
various types and can only be determined by experiment.

**3.3.2** The basic equation for the impedance $Z$ or admittance $\gamma$ of the equivalent electric circuit of the piezoelectric vibrator is given below:

$$Z = \frac{1}{\gamma} = \frac{j}{\omega C_0} \cdot \frac{\Omega - j\delta}{1 - \Omega + j\delta} \quad \ldots \quad (1)$$

**3.3.3** Equation (1) describes the relationship between the various parameters of the piezoelectric vibrator. The symbols used in equation (1) and the other essential parameters are given in Table 1. The characteristic frequencies of equation (1) are defined in Table 2.

**3.3.4** The magnitude of the impedance of the equivalent electric network ($|Z|$), its resistive component ($R_e$), its reactive component ($X_e$) and the reactance $X_1$ of the $C_1$, $L_1$, $R_1$ branch are plotted as functions of frequency in Fig. 2 for the purpose of defining the different characteristic frequencies. $|Z_m|$ and $|Z_n|$ denote minimum and maximum impedance respectively. $R_r$ and $R_a$ are the impedances at zero phase angle. These curves, however, have only qualitative character and do not represent a particular piezoelectric vibrator.




NOTE 1 — $|Z_m|$ and $|Z_n|$ denote minimum and maximum impedance. $R_r$ and $R_a$ are the impedances at zero phase angle.

NOTE 2 — The identification of different frequencies are given in Table 1.

FIG. 2 IMPEDANCE $|Z|$, RESISTANCE $R_e$, REACTANCE $X_e$ AND SERIES ARM REACTANCE $X_1$ OF A PIEZOELECTRIC VIBRATOR AS A FUNCTION OF FREQUENCY

## TABLE 1 LIST OF SYMBOLS USED FOR THE EQUIVALENT ELECTRIC CIRCUIT OF A PIEZOELECTRIC VIBRATOR

(*Clauses* 3.3.3, 4.1.2, and *Fig.* 2 *and* 3)

| Symbol | Meaning | SI Unit |
|---|---|---|
| $B_p$ | Equivalent parallel susceptance of vibrator | siemen |
| $C_0$ | Shunt (parallel) capacitance in the equivalent electric circuit | farad |
| $C_1$ | Motional capacitance in the equivalent electric circuit | farad |
| $f$ | Frequency | hertz |
| $f_a$ | Antiresonance frequency, zero susceptance | hertz |
| $f_m$ | Frequency of maximum admittance (minimum impedance) | hertz |
| $f_n$ | Frequency of minimum admittance (maximum impedance) | hertz |
| $f_p$ | Parallel resonance frequency (lossless) $= \dfrac{1}{2\pi\sqrt{L_1 \dfrac{C_1 C_0}{C_1 + C_0}}}$ | hertz |
| $f_r$ | Resonance frequency, zero susceptance | hertz |
| $f_s$ | Motional (series) resonance frequency $= \dfrac{1}{2\pi\sqrt{L_1 C_1}}$ | hertz |
| $G_p$ | Equivalent parallel conductance of vibrator | siemen |
| $L_1$ | Motional inductance in the equivalent electric circuit | henry |
| $M$ | Figure of merit of a vibrator: $M = \dfrac{Q}{r} = \dfrac{1}{\omega_s C_0 R_1}$ | dimensionless |
| $Q$ | Quality factor: $Q = \dfrac{\omega_s L_1}{R_1} = \dfrac{1}{\omega_s C_1 R_1} = r M$ | dimensionless |
| $r$ | Capacitance ratio: $r = \dfrac{C_0}{C_1}$ | dimensionless |
| $R_a$ | Impedance at zero phase angle near antiresonance | ohm |
| $R_e$ | Equivalent series resistance of vibrator | ohm |
| $R_r$ | Impedance at $f_r$ zero phase angle | ohm |
| $R_1$ | Motional resistance in the equivalent electric circuit | ohm |
| $X_e$ | Equivalent series reactance of vibrator | ohm |
| $X_0$ | Reactance of shunt (parallel) capacitance at series resonance: $X_0 = \dfrac{1}{\omega_s C_0}$ | ohm |

(*Continued*)

**TABLE 1 LIST OF SYMBOLS USED FOR THE EQUIVALENT ELECTRIC CIRCUIT OF A PIEZOELECTRIC VIBRATOR** — *Contd*

| Symbol | Meaning | SI Unit |
|---|---|---|
| $X_1$ | Reactance of motional series arm of vibrator: $X_1 = \omega L_1 - \frac{1}{\omega C_1}$ | ohm |
| $Y$ | Admittance of vibrator: $Y = G_p + jB_p = \frac{1}{Z}$ | siemen |
| $Y_m$ | Maximum admittance of vibrator | siemen |
| $Y_n$ | Minimum admittance of vibrator | siemen |
| $Z$ | Impedance of vibrator: $Z = R_e + jX_e$ | ohm |
| $Z_m$ | Minimum impedance of vibrator | ohm |
| $Z_n$ | Maximum impedance of vibrator | ohm |
| $\lvert Z \rvert$ | Absolute value of impedance of vibrator: $Z = \sqrt{R_e^2 + X_e^2}$ | ohm |
| $\lvert Z_m \rvert$ | Absolute value of impedance at $f_m$ (minimum impedance) | ohm |
| $\lvert Z_n \rvert$ | Absolute value of impedance at $f_n$ (maximum impedance) | ohm |
| $\delta$ | Normalized damping factor: $\delta = \omega C_0 R_1$ | dimensionless |
| $\Omega$ | Normalized frequency factor: $\Omega = \frac{f^2 - f_s^2}{f_r^2 - f_s^2}$ | dimensionless |
| $\omega$ | Circular (angular) frequency: $\omega = 2 \pi f$ | radian/second |
| $\omega_s$ | Circular frequency at motional resonance: $\omega_s = 2 \pi f_s$ | radian/second |

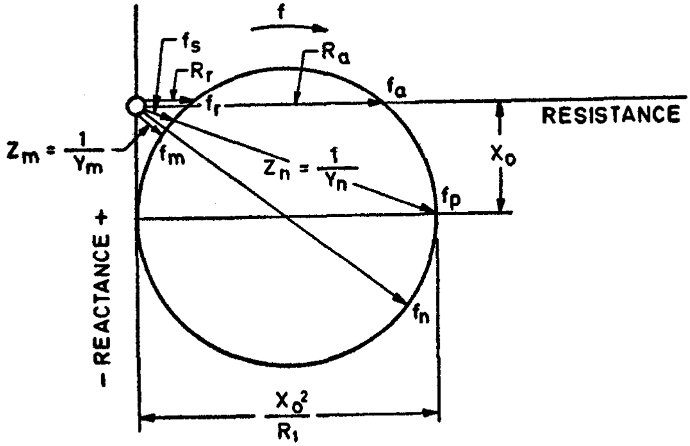
**3.3.5** For further clarification, the impedance and admittance circles of a piezoelectric vibrator are reproduced in Fig. 3. However, the circle representation of the impedance or admittance of a piezoelectric vibrator is valid only if the circle diameter of the admittance diagram is large compared with the change of $2 \pi f C_0$ in the resonance range or if $r \ll Q^2$, which is fulfilled in most vibrators. If the latter conditions are not fulfilled, the admittance curve shows a cissoidal character. In this standard, it is assumed that the impedance (or admittance) of the vibrator can be represented by a circle diagram. Table 3 gives data for $Q$, $r$ and $Q^2/r$ for various types of vibrators indicating that this assumption is valid for all practical cases.

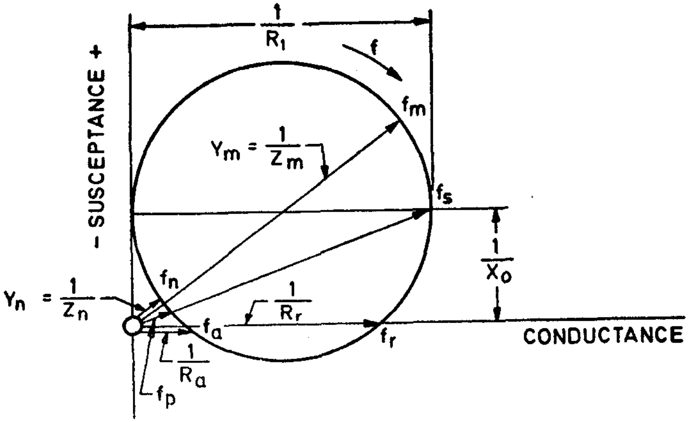
**3.3.6** It is necessary to make approximations in deriving practical equations for general use. It is the error of these approximations, in addition

to the errors of instrumentation, that govern the overall accuracy of the experimentally derived parameters.

**3.3.7** As a first approximation sufficient for many practical purposes, the following assumptions can be made:

$$f_m = f_r = f_s \text{ and } f_a = f_n = f_p$$




NOTE — The symbols conform with those in Table 1 and Fig. 2.

FIG. 3 IMPEDANCE AND ADMITTANCE DIAGRAM OF A PIEZOELECTRIC VIBRATOR

**3.3.8** More exact relations between the characteristic frequencies $f_m$, $f_r$, $f_a$, $f_p$, $f_n$ and the series resonance frequency $f_s$ of a vibrator valid for the figure of merit $M > 10$ and the capacitance ratio $r > 10$ are shown in Table 4. These relationships have been derived by various authors under the assumption that $M \gg 1$.

**3.3.9** The separation between parallel and series resonance frequencies is given below:

$$\frac{f_p^2 - f_s^2}{f_s^2} = \frac{C_1}{C_0} = \frac{1}{r} \qquad \text{... (2)}$$

The approximation

$$\frac{f_p - f_s}{f_s} = \sqrt{1 + r^{-1}} - 1$$

$$= \frac{1}{2r}\left(1 - \frac{1}{4r} + \ldots\ldots\right) \approx \frac{1}{2r}$$

$$= \frac{1}{2}\frac{C_1}{C_0} \qquad \text{... (3)}$$

can be used for larger value of $r$ (for example, when $r$ is greater than 25 the error is less than 1 percent).

## 4. TRANSMISSION CIRCUIT METHOD OF MEASURING THE PARAMETERS OF THE EQUIVALENT ELECTRIC CIRCUIT

### 4.1 General

**4.1.1** This method is based on measuring the frequency and impedance at maximum transmission (maximum transfer impedance) of a $\pi$-network containing the equivalent electric circuit of the vibrator under test in the series branch, as shown in Fig. 4. The frequency $f_{mT}$ at maximum transmission (maximum output voltage) is measured both with and without the capacitance $C_L$ in series with the vibrator. From these measurements, the motional resonance frequency $f_s$ and the motional capacitance $C_1$ of the vibrator can be determined. The value for $R_1$ can be obtained by substitution of a resistance $R_{st}$ in place of the vibrator to obtain the same output voltage.

**4.1.2** All symbols describing the transmission method are given in Tables 1 and 5.

**4.1.3** Table 6 contains, in the second column, a compilation of the expressions for $R_1$ and, in the third column, the exact deviation of the frequency at minimum transfer admittance $f_{mT}$ from the motional resonance frequency $f_s$ of the vibrator. The exact solutions shown in Table 6 can be simplified if the assumptions mentioned in the left-hand column of the table are made. When the parallel inductance $L_0$ is not used, then $b = 1$.

## TABLE 2 SOLUTIONS FOR THE VARIOUS CHARACTERISTIC FREQUENCIES

(*Clause* 3.3.3)

| Characteristic Frequency | Meaning | Condition | Constituent Equation for Frequency | Root |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| $f_m$ | Frequency of maximum admittance (minimum impedance) | $\frac{d\lvert Z\rvert}{d\omega} = 0$ | $(\Omega^2+\delta^2)^2-2\delta^2(\Omega+r)$ $-2\Omega r(1-\Omega)-\Omega^2 = 0$ | Lower* |
| $f_s$ | Motional (series) resonance frequency | $X_1 = 0$ | $\Omega=0$ | |
| $f_r$ | Resonance frequency | $X_e = B_p = 0$ | $\Omega(1-\Omega)-\delta^2 = 0$ | Lower |
| $f_a$ | Antiresonance frequency | $X_e = B_p = 0$ | $\Omega(1-\Omega)-\delta^2 = 0$ | Upper |
| $f_p$ | Parallel resonance frequency (lossless) | $X_e\big\rvert_{R_1 = 0} = \infty$ | $\Omega = 1$ | |
| $f_n$ | Frequency of minimum admittance (maximum impedance) | $\frac{d\lvert Z\rvert}{d\omega} = 0$ | $(\Omega^2+\delta^2)^2-2\delta^2(\Omega+r)$ $-2\Omega r(1-\Omega)-\Omega^2 = 0$ | Upper* |

*Refers to real roots; complex roots to be disregarded.

## TABLE 3 MINIMUM VALUES FOR THE RATIO $Q^2/r$ TO BE EXPECTED FOR VARIOUS TYPES OF PIEZOELECTRIC VIBRATORS

(*Clause* 3.3.5)

| Type of Piezoelectric Vibrator | $Q=Mr$ | $r$ | $Q^2/r$ min |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| Piezoelectric ceramics | 90 - 500 | 2 - 40 | 200 |
| Water-soluble piezoelectric crystals | 200 - 50 000 | 3 - 500 | 80 |
| Quartz | $10^4$ - $10^7$ | 100 - 50 000 | 2 000 |

TABLE 4 APPROXIMATE RELATIONS BETWEEN THE CHARACTERISTIC FREQUENCIES AND THE SERIES RESONANCE FREQUENCY $f_s$ OF A PIEZOELECTRIC VIBRATOR

(*Clause* 3.3.8)

| CHARACTERISTIC FREQUENCY | FIRST APPROXIMATION | | SECOND APPROXIMATION | |
|---|---|---|---|---|
| | $\frac{f}{f_s}$ | Deviation $\frac{\Delta f}{f_s}$ from more precise value | $\frac{f}{f_s}$ | Deviation $\frac{\Delta f}{f_s}$ from more precise value |
| (1) | (2) | (3) | (4) | (5) |
| $f_m$ | $\frac{f_m}{f_s}=1$ | $-\frac{1}{2M^2r}$ | $\frac{f_m}{f_s}=\sqrt{1+\frac{1}{2r}\left(1-\sqrt{1+\frac{4}{M^2}}\right)}$ | $\frac{1}{2M^4r^2}$ |
| $f_r$ | $\frac{f_r}{f_s}=1$ | $\frac{1}{2M^2r}$ | $\frac{f_r}{f_s}=\sqrt{1+\frac{1}{2r}\left(1-\sqrt{1-\frac{4}{M^2}}\right)}$ | $\frac{1}{2M^4r^2}$ |
| $f_a$ | $\frac{f_a}{f_s}=1+\frac{1}{2r}$ | $-\frac{1}{2M^2r}\left(\frac{1}{r}+1\right)$ | $\frac{f_a}{f_s}=\sqrt{1+\frac{1}{2r}\left(1+\sqrt{1-\frac{4}{M^2}}\right)}$ | $-\frac{1}{2M^2r}\cdot\frac{1}{r}$ |
| $f_n$ | $\frac{f_n}{f_s}=1+\frac{1}{2r}$ | $\frac{1}{2M^2r}\left(\frac{1}{r}+1\right)$ | $\frac{f_n}{f_s}=\sqrt{1+\frac{1}{2r}\left(1+\sqrt{1+\frac{4}{M^2}}\right)}$ | $\frac{1}{2M^2r}\cdot\frac{1}{r}$ |
| $f_p$ | $\frac{f_p}{f_s}=1+\frac{1}{2r}$ | $-\frac{1}{8r^2}$ | $\frac{f_p}{f_s}=\sqrt{1+\frac{1}{r}}$ | 0 |

## 4.2 Transmission Measurement Circuit

**4.2.1** Figure 4 shows a schematic representation of the transmission circuit. The measuring circuit consists of a constant current source in the form of a variable frequency oscillator, the transmission network and a voltmeter. The piezoelectric vibrator is represented by its equivalent electric circuit. The network is symmetrical with respect to the input and output. The capacitances $C_T$, which shunt the terminating resistances $R_T$, represent stray element which affect the accuracy of measurements as shown in Table 6. The oscillator shall have a high degree of purity of output waveform to an extent consistent with the requirements of the individual vibrator under test (*see* Note).

NOTE — For most practical purposes, the following requirements are adequate: harmonics greater than 30 dB below the main signal, frequency stability better than $1\times10^{-6}$ and amplitude change less than 10 percent during the measurement period.

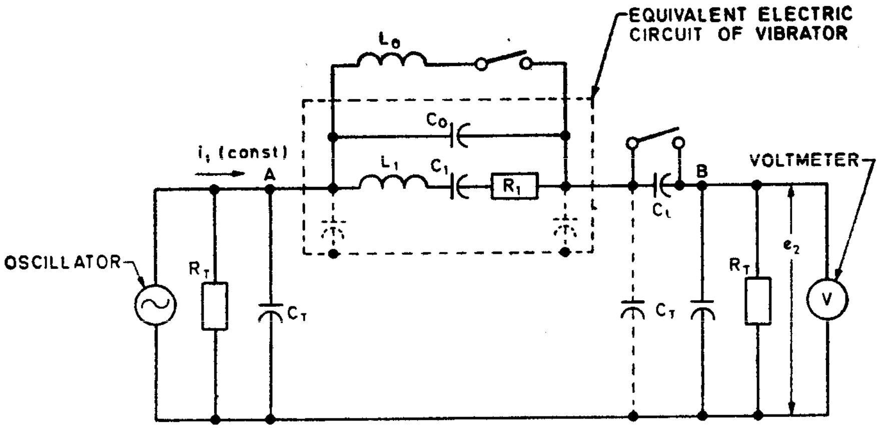



FIG. 4 SCHEMATIC DIAGRAM OF TRANSMISSION CIRCUIT METHOD

**4.2.2** The inductance $L_0$, connected across the vibrator, serves to resonate the shunt capacitance $C_0$ of the vibrator at $f_s$. This added component improves the accuracy of measurement as explained in **4.3.1**.

**4.2.3** An important source of stray capacitance to ground that shall be considered occurs at the junction of the crystal unit and the capacitor $C_L$ (Fig. 4). This stray capacitance is composed of two parts: that associated with the crystal unit under test and that associated with $C_L$. When the magnitude of these stray reactances is large compared with the magnitude of the termination impedance, the distributed capacitance to ground of the crystal unit and the distributed capacitance to ground of $C_L$ at the junction may be treated in a first approximation as being in parallel with $C_L$. While measuring the parameters of crystal units by other than the transmission method, consideration shall be given to the stray capacitance of each terminal of the crystal unit to ground. This is of course an important consideration in the use of crystal units in network and frequency control applications.

**4.2.4** The voltmeter which is placed at the output, measures the voltage $e_2$ as the frequency of the input is varied.

**4.3 Procedure for Measurement and Determination of the Parameters**

**4.3.1** *Motional Resistance* $R_1$ — The motional resistance is measured by adjusting the frequency of the oscillator to obtain maximum transmission (maximum $e_2$) of the vibrator. The resistor is adjusted to the particular value $R_{st}$ for which the maximum transmission value $e_2$ is equal to the value obtained with the vibrator. The general relation between $R_1$ and $R_{st}$ is given in equation (4) (*see* Table 6). Assuming that $C_T=0$, $R_1$ can be calculated from equation (4b) (*see* Table 6). While substituting $R_{st}$

for $R_1$ and when compensating inductance $L_0$ is not used, the error in $R_1$ will be:

$$100 \cdot \left(\frac{R_{st}}{X_0}\right)^2 \left(\frac{4\ R_T}{R_{st}} + 1\right) \text{ percent} \qquad \ldots \quad (5)$$

**4.3.1.1** Proper adjustment of $L_0$ for $b = 0$ (*see* Table 5) reduces this error to zero [*see* equation (4f) in Table 6].

**4.3.2** *Motional Capacitance $C_1$ and Inductance $L_1$* — The motional capacitance $C_1$ is determined by measuring the frequency of maximum transmission $f_{mT}$ using one or more load capacitances $C_L$ (*see* Note) connected successively in series with the vibrator (*see* Fig. 4). When $M \gg 1$, which is usually the case, the readings obtained by use of two different load capacitances $C_{L1}$ and $C_{L2}$ can be combined so that:

$$C_1 = \frac{2\triangle C_L}{f_s} \ \frac{\triangle f_1 \triangle f_2}{\triangle f} \qquad \ldots \quad (6)$$

where

$$\left.\begin{aligned} \triangle C_L &= C_{L2} - C_{L1} \\ \triangle f &= f_{sL1} - f_{sL2} \\ \triangle f_1 &= f_{sL1} - f_s \\ \triangle f_2 &= f_{sL2} - f_s \end{aligned}\right\} \qquad \ldots \quad (7)$$

and $f_{sL1}$ and $f_{sL2}$ are the motional resonance frequencies of the vibrator in series with $C_{L1}$ and the vibrator in series with $C_{L2}$ respectively. The frequencies of maximum transmission may be used in equation (7) instead of the respective motional resonance frequencies resulting in an error in $C_1$ of less than 3 percent for $(Q^2/r) \geqslant 80$. To obtain higher accuracy, the motional resonance frequencies have to be calculated from the frequencies of maximum transmission according to **4.3.3** and Table 6. As the equations in Table 6 are based on the equivalent electric circuit shown in Fig. 1, equations (8) have to be used to obtain the corresponding parameters for the combination of vibrator plus $C_L$. When a capacitance $C_L$ is in series with an electroded element, the diagram shown in Fig. 5 applies. The two circuits are equivalent having the following relationships:

$$\left.\begin{aligned} L'_1 &= L_1 \left(1 + \frac{C_0}{C_L}\right)^2 \\ C'_1 &= C_1 \frac{1}{\left(1 + \frac{C_0}{C_L}\right)^2 \left(1 + \frac{C_1}{C_0 + C_L}\right)} \\ R'_1 &= R_1 \left(1 + \frac{C_0}{C_L}\right)^2 \\ C'_0 &= \left(\frac{C_0\ C_L}{C_0 + C_L}\right) \end{aligned}\right\} \qquad \ldots \quad (8)$$

NOTE — *See* **4.2** for the effect of stray capacitance on $C_L$.

**4.3.2.1** When the frequencies of maximum transmission are measured using different values for $C_L$, it is important to maintain the current through the vibrator constant to within 10 percent as indicated by the voltmeter.

**4.3.2.2** The inductance $L_1$ follows from:

$$L_1 = (\omega_s^2 C_1)^{-1} \qquad \dots \quad (9)$$

when the values for $\omega_s^2$ and $C_1$ are known.

**4.3.3** *Motional Resonance Frequency $f_s$* — The frequency of the oscillator is adjusted for maximum transmission with the piezoelectric vibrator inserted in the network shown in Fig. 4. This is the frequency of maximum transmission $f_{mT}$. At the first approximation, $f_{mT}$ is equal to the frequency of minimum impedance $f_m$ and the motional resonance frequency $f_s$ of the vibrator. If higher accuracy for $f_s$ is required, Table 6 should be consulted which gives the relationship between $f_{mT}$ and $f_s$ as the ratio:

$$\frac{f_{mT}^2}{f_s^2} - 1$$

for different degrees of approximation as a function of the network parameters. When the shunting inductance $L_0$ is omitted, equation (10b) in Table 6 becomes

$$\frac{f_{mT}^2}{f_s^2} - 1 = \frac{1}{r}\left[1 + \frac{2}{M^2\left(1 + \frac{4\,R_T}{M^2 R_1}\right)\left\{1 - \sqrt{1 + \frac{4}{M^2\left(1 + \frac{4\,R_T}{M^2 R_1}\right)^2}}\right\}}\right] \qquad \dots \quad (11a)$$

**4.3.3.1** When $M^2 \gg 1$ is fulfilled for the resonator, the general formula 10 reduces to:

$$\frac{f_{mT}^2}{f_s^2} - 1 \approx \frac{-1}{M^2 r}\left(\frac{4\,R_T}{R_1} + 1\right) \qquad \dots \quad (11b)$$

and since the right-hand side of equation (11b) is usually much smaller than unity, it further reduces to:

$$\frac{f_{mT} - f_s}{f_s} = \frac{\Delta f}{f_s} \approx \frac{-1}{2\,M^2 r}\left(\frac{4\,R_T}{R_1} + 1\right) \qquad \dots \quad (11c)$$

**4.3.3.2** In most instances, $M^2 \gg 1$, and the approximate equation (11c) is satisfactory. When this condition is not fulfilled, the exact formula (11a) shall be used.

**4.3.4** *Network Requirements*

**4.3.4.1** The accuracy of the results increases as the following conditions

are fulfilled:

a) Stray capacitance $C_{A-B}$ between terminals $A$ and $B$ is low compared to vibrator capacitance $C_0$ ($C_0 \gg C_{A-B}$).

b) Reactance of stray capacitance $C_{A-B}$ is high compared to series resistance $R_1$ ($|X_{A-B}| \gg R_1$).

c) The reactance of leads connecting vibrator is low compared to reactance of $C_0$.

**4.3.4.2** In the case of vibrators with low figure of merit $M$, it is advisable to use a shunting coil $L_0$ connected in parallel with the vibrator. If the combination $L_0C_0$ is tuned to the motional resonance frequency $f_s$ of the vibrator, $b=0$ and the measurement is simplified. In Table 6, equations (4a) and (10a) supply resistance and frequency values for this condition. It is seen that when $b=0$ and $M_T \gg 1$ [condition (a) in **4.3.4.1**], then $f_{mT}=f_s$ and $R_{st}=R_1$.

**4.3.5** *Shunt Capacitance* $C_0$

**4.3.5.1** The shunt capacitance $C_0$ of the equivalent electric circuit of a vibrator is slightly smaller than the measured value for a free piezoelectric element and slightly greater than the measured value for a piezoelectric element in clamped condition. The exact value of the dielectric permittivity depends upon the mode of vibration. This has to be considered when greater accuracy is required.

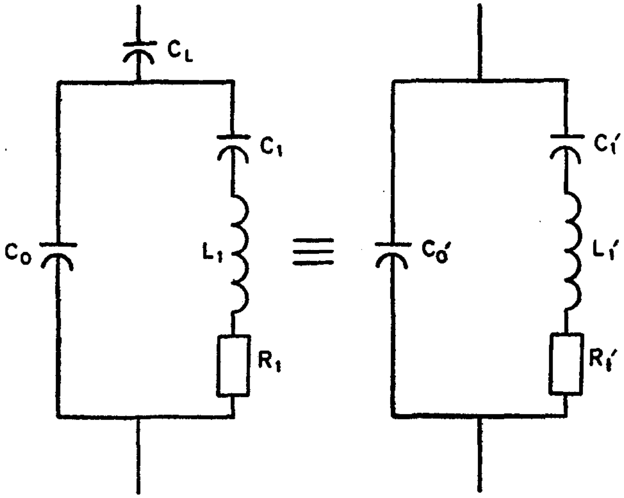



FIG. 5 EQUIVALENT ELECTRIC CIRCUIT OF A PIEZOELECTRIC VIBRATOR WITH A SERIES (LOAD) CAPACITANCE $C_L$

**TABLE 5 LIST OF SYMBOLS USED IN TRANSMISSION NETWORK**

(*Clauses* 4.1.2 *and* 4.3.1.1)

| Symbol | Meaning | SI Unit |
|---|---|---|
| $b$ | Normalized compensation factor: $1 - \frac{1}{4\,\pi^2 f_s^2\, C_0\, L_0}$ | dimensionless |
| $B$ | Normalized admittance factor | dimensionless |
| $C$ | Normalized admittance factor | dimensionless |
| $C_{A-B}$ | Stray capacitance between the terminals A-B (Fig. 4) | farad |
| $C_L$ | Load capacitance | farad |
| $C_T$ | Shunt capacitance terminating transmission circuit | farad |
| $C_{L1}$ | Load capacitance | farad |
| $C_{L2}$ | Load capacitance | farad |
| $e_2$ | Output voltage of transmission network | volt |
| $f_{mT}$ | Frequency of maximum transmission | hertz |
| $f_{sL1}$ | Motional resonance frequency of combination of vibrator and $C_{L1}$ | hertz |
| $f_{sL2}$ | Motional resonance frequency of combination of vibrator and $C_{L2}$ | hertz |
| $i_1$ | Input current to transmission network | ampere |
| $L_0$ | Compensation inductance shunting vibrator | henry |
| $M_T$ | Figure of merit of transmission network termination: $M_T = \frac{1}{2\,\pi f_s\, C_T\, R_T} = \frac{X_T}{R_T}$ | dimensionless |
| $R_T$ | Shunt resistance termination of transmission network | ohm |
| $R_{st}$ | Standard resistor | ohm |
| $S$ | Detector sensitivity (smallest detectable current change/current) | dimensionless |
| $x$ | Normalized frequency factor: $x = \frac{f^2}{f_s^2} - 1 = \frac{\Omega}{r}$ | dimensionless |
| $X_{A-B}$ | Reactance of stray capacitance $C_{A-B}$ | ohm |
| $X_T$ | Reactance of $C_T$ at the motional resonance frequency: $X_T = \frac{1}{2\,\pi f_s\, C_T}$ | ohm |
| $x_{mT}$ | Normalized frequency factor at the frequency of maximum transmission | dimensionless |
| $\Delta C_L$ | $\Delta C_L = C_{L2} - C_{L1}$ | farad |
| $\Delta f$ | $\Delta f = f_{sL1} - f_{sL2}$ | hertz |
| $\Delta f_1$ | $\Delta f_1 = f_{sL1} - f_s$ | hertz |
| $\Delta f_2$ | $\Delta f_2 = f_{sL2} - f_s$ | hertz |

## TABLE 6 RELATIONSHIP BETWEEN MEASURED AND FUNDAMENTAL VALUES

(*Clauses* 4.1.3, 4.2.1, 4.3.1, 4.3.1.1, 4.3.2, 4.3.3, 4.3.4.2 *and* 4.4.3)

| | $R_1 =$ | $\dfrac{f^2_{mT}}{f_s^2} - 1 =$ |
|---|---|---|
| Complete solution | $\dfrac{R_T(2-\nu)}{\dfrac{C_0^2}{C_T^2}\dfrac{b^2\nu^2}{1+M_T^2} - \dfrac{4b}{M_T^2}\dfrac{C_0}{C_T}\left(b\dfrac{C_0}{C_T}+1\right) - \left(1+\dfrac{1}{M_T^2}\right)}$ ... (4)<br>where<br>$\nu = \sqrt{\left\{2+\dfrac{R_{st}}{R_T}\left(1+\dfrac{1}{M_T^2}\right)\right\}^2 + \dfrac{4}{M_T^2}}$ | $\dfrac{1}{rb} + \dfrac{\dfrac{b}{rM^2}\left(1+C+\dfrac{2}{M_T^2}\right)}{B-1+\left(1+\dfrac{1}{M_T^2}\right)\left[1-\sqrt{\dfrac{1}{1+\dfrac{1}{M_T^2}}\left\{B^2+\dfrac{4b^2}{M^2}C+\dfrac{1}{M_T^2}\left(1+\dfrac{4b^2}{M^2}\right)\right\}}\right]}$ ... (10)<br>where<br>$B = 1+\dfrac{4b^2}{M^2}\dfrac{R_T}{R_1}$<br>$C = 1+\dfrac{4b}{M\,M_T}\dfrac{R_T}{R_1}$ |
| $b = 0$ | $\dfrac{R_T\left[\sqrt{\left\{2+\dfrac{R_{st}}{R_T}\left(1+\dfrac{1}{M_T^2}\right)\right\}^2+\dfrac{4}{M_T^2}}-2\right]}{1+\dfrac{1}{M_T^2}}$ ... (4a) | $\dfrac{2R_T}{r\,M\,M_T\,R_1\left(1+\dfrac{1}{M_T^2}\right)}$ ... (10a) |
| $C_T = 0$ | $\dfrac{R_{st}}{1-\dfrac{R_{st}^2}{X_0^2}b^2\left(4\dfrac{R_T}{R_{st}}+1\right)}$ ... (4b) | $\dfrac{1}{rb} + \dfrac{2b}{rM^2B\left(1-\sqrt{1+\dfrac{4b^2}{M^2B^2}}\right)}$ ... (10b) |

| | $R_1 =$ | $\dfrac{f^2_{m\mathrm{T}}}{f_s^2} - 1 =$ |
|---|---|---|
| $R_\mathrm{T} = \infty$ | $\dfrac{R_{st}\sqrt{1+\left(\dfrac{2X_\mathrm{T}}{R_{st}}\right)^2}}{1+4b\dfrac{C_0}{C_\mathrm{T}}-\dfrac{R_{st}^2}{X_0^2}b^2}$ ... (4c) | $\dfrac{1}{rb}+\dfrac{2b\left(2b\dfrac{C_0}{C_\mathrm{T}}+1\right)}{rM^2\left\{1-\sqrt{\dfrac{4b^2}{M^2}\left(1+2b\dfrac{C_0}{C_\mathrm{T}}\right)^2+1}\right\}}$ ... (10c) |
| $\dfrac{4}{M_\mathrm{T}^2} \ll 1$ | $\dfrac{R_{st}}{1+\dfrac{4b}{M_\mathrm{T}^2}\dfrac{C_0}{C_\mathrm{T}}-\dfrac{b^2}{M_\mathrm{T}^2}\left(\dfrac{R_{st}}{R_\mathrm{T}}\dfrac{C_0}{C_\mathrm{T}}\right)^2\left(1+4\dfrac{R_\mathrm{T}}{R_{st}}\right)}$ ... (4d) | $\dfrac{2}{M^2 r}\dfrac{C_\mathrm{T}}{C_0}\dfrac{R_\mathrm{T}^2}{R_1^2}-\dfrac{4b}{M^2}\dfrac{R_\mathrm{T}}{R_1 r}-\dfrac{b}{M^2 r}$ is valid for $b^2/M^2 \ll 1$ ... (10d) |
| $b = 0$; $R_\mathrm{T} = \infty$ | $R_{st}\sqrt{1+\left(\dfrac{2X_\mathrm{T}}{R_{st}}\right)^2}$ ... (4e) | $2\dfrac{C_1}{C_\mathrm{T}}$ ... (10e) |
| $b = 0$ $C_\mathrm{T} = 0$ | $R_{st}$ ... (4f) | 0 ... (10f) |

**4.3.5.2** There is no direct method for measuring $C_0$ precisely. However, in nearly all practical cases it is adequate to regard as $C_0$ the mean value of the shunt capacitances $C_{01}$ and $C_{02}$ of the resonator obtained at two frequencies, equidistant above and below the resonance frequency and sufficiently removed from the latter for the impedance to be independent of any response. $C_{01}$ and $C_{02}$ can be measured by means of an impedance bridge or a Q-meter.

**4.3.5.3** It should be noted that $C_0$ is the shunt capacitance between the two electrodes of the resonator. As pointed out in **4.2**, the capacitances of both of the electrodes to ground are important elements in many network and frequency control applications. Proper use of the transmission circuit method for the measurement of $C_1$ requires knowledge of at least the capacitance to ground of that electrode of the resonator which is connected to $C_L$ (*see* Fig. 4).

**4.3.5.4** Therefore, in the general case, it is necessary to consider the crystal unit as a three-terminal network and to evaluate $C_0$ and the stray capacitances of the two electrodes to ground from open- and short-circuit measurements according to the techniques customarily employed when dealing with two-port devices.

**4.3.5.5** The crystal enclosure remains at ground potential during the entire series of measurements required for evaluation of the resonator parameters. For this purpose, it may be found desirable to provide glass-enclosed crystal units with metal shells.

## 4.4 Numerical Examples

**4.4.1** The mean deviation of $f_s$ from its true values, due to detector sensitivity $S$ alone, is given by:

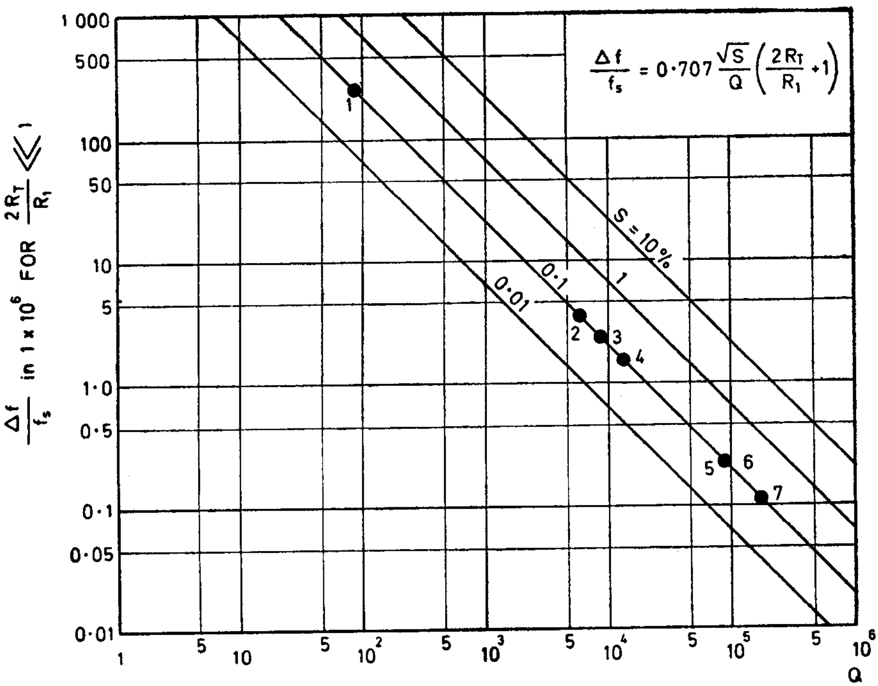
$$x \approx \frac{\Delta f}{f_s} = \frac{1}{\sqrt{2}}\left(\frac{2\,R_T}{R_1}+1\right)\frac{\sqrt{S}}{Q} \qquad \dots \quad (12)$$

**4.4.2** This equation is valid if $4/M_T^2 \ll 1$ and $b^2/M^2 \ll 1$. The magnitude of the mean deviation is plotted in Fig. 6 for various values of $S$ as a function of $Q$.

**4.4.3** The equations in Table 6 give the corrections necessary to obtain $R_1$ from $R_{st}$ and $f_s$ from $f_{mT}$ respectively in terms of the vibrator and network parameters. When the assumptions leading to the simplified relations [equations (5) and (11c)] are met, the magnitude of these corrections can be obtained from the graphs shown in Fig. 7 and 8.

**4.4.4** The solid lines in these graphs further assume that 2 $(R_T/R_1) \ll 1$ (Fig. 6), $\dfrac{4(R_T)}{R_{st}} \ll 1$ (Fig. 7), and $\dfrac{4(R_T)}{R_1} \ll 1$ (Fig. 8). The ordinate values

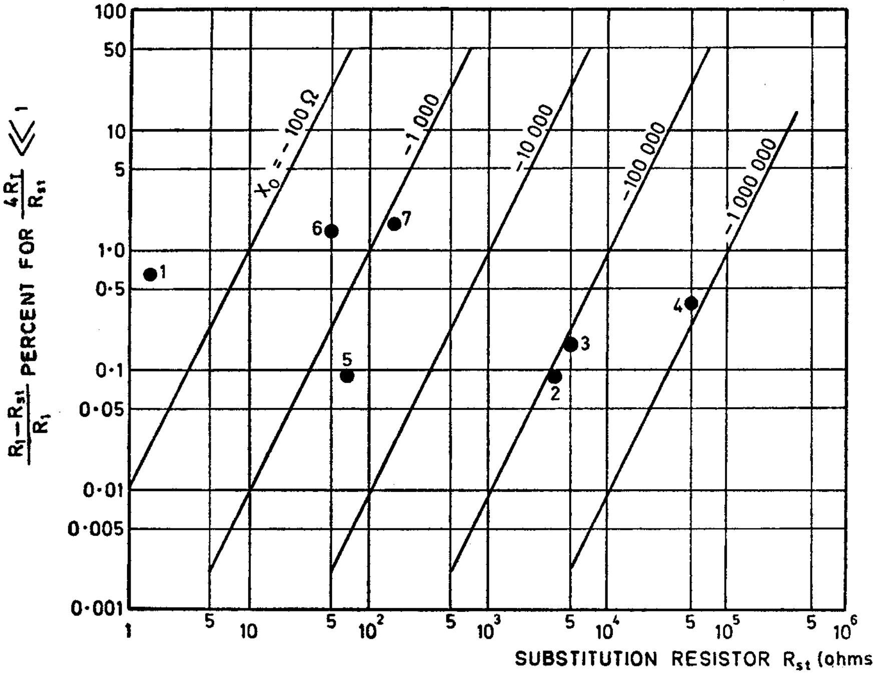
of the graphs (Fig. 6 to 8) can be modified easily when these relationships are not fulfilled. The examples shown in the graphs refer to the vibrators listed in Fig. 6 and illustrate that $R_1$ generally differs from $R_{st}$ by less than 2 percent (Fig. 7) and that, excluding ceramics, the differences between $f_s$ and $f_{mT}$ are of the order $1 \times 10^{-6}$ (Fig. 8). Shunt coils were not used for these measurements.




*Examples:*

1. Barium titanate longitudinal mode
2. Quartz CT element 300 kHz
3. Quartz DT element 300 kHz
4. Quartz NT element 100 kHz
5. Quartz AT 3rd overtone 15 MHz
6. Quartz AT 5th overtone 70 MHz
7. Quartz AT 5th overtone 23·3 MHz

FIG. 6 MEAN DEVIATION $\frac{\Delta f}{f_s}$ DUE TO VOLTMETER SENSITIVITY $S$

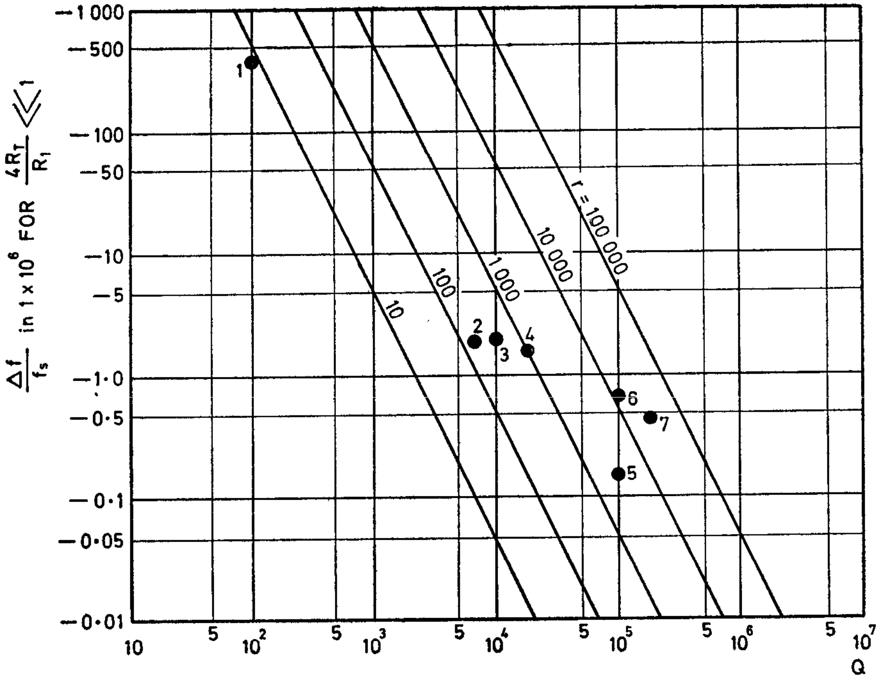



*Examples:*

1. Barium titanate longitudinal mode
2. Quartz CT element 300 kHz
3. Quartz DT element 300 kHz
4. Quartz NT element 100 kHz
5. Quartz AT 3rd overtone 15 MHz
6. Quartz AT 5th overtone 70 MHz
7. Quartz AT 5th overtone 23·3 MHz

NOTE : $1 - \frac{R_{st}}{R_1} = \left(\frac{R_{st}}{X_0}\right)^2 \left(\frac{4\ R_T}{R_{st}} + 1\right)$

FIG. 7 RESISTANCE MEAN DEVIATION




*Examples :*

1. Barium titanate longitudinal mode
2. Quartz CT element 300 kHz
3. Quartz DT element 300 kHz
4. Quartz NT element 100 kHz
5. Quartz AT 3rd overtone 15 MHz
6. Quartz AT 5th overtone 70 MHz
7. Quartz AT 5th overtone 23·3 MHz

NOTE : $\frac{\Delta f}{f_s} = - \frac{r}{2Q^2}\left(1 + \frac{4R_T}{R_1}\right)$

FIG. 8 MEAN DEVIATION $\frac{\Delta f}{f_s}$ DUE TO VIBRATOR AND CIRCUIT PARAMETERS

**INDIAN STANDARDS**

**ON**

**PIEZOELECTRIC DEVICES FOR FREQUENCY CONTROL AND SELECTION**

IS :

1885 (Part V)-1965 Electrotechnical vocabulary: Part V Quartz crystals

1885 (Part XXXIII)-1972 Electrotechnical vocabulary: Part XXXIII Piezoelectric filters

2916 (Part I)-1969 Quartz crystal units used in oscillators: Part I General requirements and tests

2916 (Part II)-1972 Quartz crystal units used in oscillators: Part II Type AA-01

2916 (Part III)-1972 Quartz crystal units used in oscillators: Part III Type AA-02

2916 (Part IV)-1972 Quartz crystal units used in oscillators: Part IV Type AA-03

2916 (Part V)-1972 Quartz crystal units used in oscillators: Part V Type AA-04

2916 (Part VI)-1972 Quartz crystal units used in oscillators: Part VI Type AA-05

2916 (Part VII)-1975 Quartz crystal units used in oscillators: Part VII Type AA-06

2935-1964 Guide for use of quartz oscillator crystals

4570-1968 Crystal holders

5575 (Part I)-1970 Temperature control devices for quartz crystal units: Part I General requirements and tests

6133 (Part I)-1971 Piezoelectric filter for use in telecommunication and measuring equipment: Part I General requirements and tests

6537-1972 Guide to the use of temperature control devices for crystal units

7410 (Part I)-1974 Guide to the use of piezoelectric filters: Part I Quartz crystal filters

7410 (Part II)-1975 Guide to the use of piezoelectric filters: Part II Piezoelectric ceramic filters

7957-1976 Basic method for the measurement of resonance frequency and equivalent series resistance of quartz crystal units by zero phase technique in a $\pi$-network

7962-1975 Methods of measurement for piezoelectric vibrators operating over the frequency range up to 30 MHz

8011-1976 Methods for the measurement of frequency and equivalent resistance of unwanted resonances of filter crystal units

## PUBLICATIONS OF INDIAN STANDARDS INSTITUTION

### INDIAN STANDARDS

Over 8 500 Indian Standards covering various subjects have been issued so far. Of these, the standards belonging to the Electrotechnical Group fall under the following categories:

- Acoustics and acoustical measurement
- Automobile electrical equipment
- Batteries
- Cinematographic equipment
- Conductors and cables
- Domestic electrical appliances
- Electric welding equipment
- Electrical installations, codes of practice
- Electrical instruments
- Electron tubes and valves
- Electronic components
- Electronic equipment
- Environmental testing procedures
- Fans
- Flameproof electrical equipment
- High voltage techniques
- Illuminating engineering
- Instrument transformers
- Insulating materials
- Insulators and accessories
- Integrating meters
- Lamps and lamp accessories
- Lifts and escalators
- Lightning arresters
- Motors and generators
- Nomenclature and symbols
- Power capacitors
- Power converters
- Relays
- Rotating machinery
- Semiconductor devices
- Switchgear and controlgear
- Transformers and reactors
- Winding wires
- Wiring accessories
- Unclassified

### OTHER PUBLICATIONS

| | |
|---|---|
| ISI Bulletin (Published Every Month) | |
| Single Copy ... ... ... ... ... | Rs 4·00 |
| Annual Subscription ... ... ... ... ... | Rs 36·00 |
| Standards: Monthly Additions | |
| Single Copy ... ... ... ... ... | Re 0·30 |
| Annual Subscription ... ... ... ... ... | Rs 3·00 |
| Annual Reports (from 1948-49 Onwards) ... ... ... | Rs 2·00 to 6·00 |
| ISI Handbook, 1975 ... ... ... ... ... | Rs 30·00 |

---


### INDIAN STANDARDS INSTITUTION

Manak Bhavan, 9 Bahadur Shah Zafar Marg, NEW DELHI 110002

**Telephone : 27 01 31 ( 20 lines )** **Telegrams : Manaksanstha**

| *Regional Offices:* | | Telephone |
|---|---|---|
| Western : Novelty Chambers, Grant Road | BOMBAY 400007 | 37 97 29 |
| Eastern : 5 Chowringhee Approach | CALCUTTA 700072 | 23-08 02 |
| Southern : C.I.T. Campus, Adyar | MADRAS 600020 | 41 24 42 |
| *Branch Offices:* | | |
| 'Pushpak', Nurmohamed Shaikh Marg, Khanpur | AHMADABAD 380001 | 2 03 91 |
| 'F' Block, Unity Bldg, Narasimharaja Square | BANGALORE 560002 | 2 76 49 |
| Ahimsa Bldg, SCO 82-83, Sector 17C | CHANDIGARH 160017 | 2 83 20 |
| 5-8-56/57 Nampally Station Road | HYDERABAD 500001 | 4 57 11 |
| 117/418 B Sarvodaya Nagar | KANPUR 208005 | 82 72 |
| B.C.I. Bldg (3rd Floor), Gandhi Maidan East | PATNA 800004 | 5 36 55 |
| Hantex Bldg (2nd Floor), Rly Station Road | TRIVANDRUM 695001 | 32 27 |


Printed at Sree Saraswaty Press Ltd., Calcutta 700 009